

# The rate of radiative recombination in the nitride semiconductors and alloys

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The radiative recombination rates of free carriers and lifetimes of free excitons have been calculated in the wide band gap semiconductors GaN, InN, and AlN of the hexagonal wurtzite structure, and in their solid solutions  $\text{Ga}_x\text{Al}_{1-x}\text{N}$ ,  $\text{In}_x\text{Al}_{1-x}\text{N}$  and  $\text{Ga}_x\text{In}_{1-x}\text{N}$  on the base of existing data on the energy band structure and optical absorption in these materials. We determined the interband matrix elements for the direct optical transitions between the conduction and valence bands, using the experimental photon energy dependence of absorption coefficient near the band edge. In our calculations we assumed that the material parameters of the solid solutions (the interband matrix element, carrier effective masses, and so on) could be obtained by a linear interpolation between their values in the alloy components. The temperature dependence of the energy gap was taken in the form proposed by Varshni [Physica **34**, 149 (1967)]. The calculations of the radiative recombination rates were performed in a wide range of temperature and alloy compositions. © 1999 American Institute of Physics. [S0021-8979(99)06917-0]

## I. INTRODUCTION

Nowadays, the nitride semiconductors such as GaN, AlN, and InN attract a considerable attention due to their outstanding physical, chemical, and mechanical properties and also because of the recent progress in technology that allowed to produce high quality nitride films with the help of metal-organic chemical vapor deposition (MOCVD) and molecular-beam epitaxy (MBE) (for a recent review, see Ref. 1). The attractive properties of the nitrides include high heat conductivity, hardness, chemical stability, and high luminescence intensity. These wide gap semiconductors are very promising materials for light emitting diodes and semiconductor lasers operating in a wide spectral interval from ultraviolet to green and even orange<sup>2</sup> because their solid solutions may have energy gap varying from 2 eV in InN to 6.2 eV in AlN. Important material characteristics for luminescence devices are the rates of different electron-hole recombination processes. However, not much is known at the moment about the intensities of these processes in the nitrides. In this work, we concentrate on the calculation of the radiative band-band recombination rate in GaN, InN, AlN, and their binary alloys  $\text{Ga}_x\text{Al}_{1-x}\text{N}$ ,  $\text{In}_x\text{Al}_{1-x}\text{N}$ , and  $\text{Ga}_x\text{In}_{1-x}\text{N}$  on the basis of the experimental absorption data that are present in the literature,<sup>3–10</sup> calculated electron energy band dispersion laws,<sup>11–14</sup> and temperature dependence of the band gaps in GaN, InN, and AlN.<sup>15–17</sup> In addition we calculated lifetimes of free excitons in the main nitride semiconductors.

## II. ENERGY SPECTRA OF THE NITRIDE SEMICONDUCTORS

We consider more common and popular hexagonal phase of the nitrides. All of them belong to the crystal class

$C_{6v}$ . Their conduction bands are nondegenerate, and their electron states originate from atomic  $s$  functions. If one first neglects the relatively small spin-orbit interaction, then in the  $\Gamma$  point of the Brillouin zone the electron wave functions transform according to  $\Gamma_1$ , the unit representation of  $C_{6v}$ . The valence band is complicated and consists of two branches. One of them transforms according to  $\Gamma_1$  whereas the other is degenerate and forms the two-dimensional representation  $\Gamma_6$ . If spin-orbit interaction is taken into account,  $\Gamma_6$  further splits into two bands,  $\Gamma_7^v$  and  $\Gamma_9^v$ , and  $\Gamma_1$  in both conduction and valence bands turn into  $\Gamma_7^c$ :  $\Gamma_1^{c,v} \rightarrow \Gamma_7^{c,v}$  (see Refs. 18 and 19). However, the spin-orbit splitting manifests itself along  $k_x$  and  $k_y$  strongly, but faint at the  $\Gamma$  point and along  $k_z$ ,  $z$  being the direction of the hexagonal axis  $c$  which usually coincides with the normal to the film.

According to the results of Refs. 11–13, 18, 20–22, the order of levels in the valence band of the nitrides is different: in GaN and InN,  $\Gamma_9^v(\Gamma_6)$  branch lies above  $\Gamma_7^v(\Gamma_1)$ , whereas in AlN it lies below  $\Gamma_7^v(\Gamma_1)$  (for more details, see Ref. 18). The symmetry of the electron wave functions in different bands and band branches leads to the following selection rules for the radiative transitions: for the transition from  $\Gamma_7^c$  to  $\Gamma_7^v$ , only  $z$  component of the transition matrix element differs from zero,<sup>4,22</sup> and correspondingly, the emitted photon is polarized along  $z$  axis. On the contrary, for the transition from  $\Gamma_7^c$  to  $\Gamma_9^v$ , the  $z$  component of the transition matrix element equals zero,<sup>22</sup> and the polarization direction of the emitted photon is perpendicular to  $z$  axis.

## III. RADIATIVE RECOMBINATION RATE: THE METHOD OF CALCULATION

Usually, the Shockley–van Roosbroeck formula<sup>23</sup> is used for the calculation of the radiative recombination intensity, which allows one to calculate the transition rate if the

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spectral dependence of the absorption coefficient is known. However, this is not the case in the nitrides where optical absorption has been measured only in a small vicinity of the band edge, so another method is needed that would allow to express the recombination rate through material parameters.

By definition, the spontaneous radiative recombination rate  $R_s$  is the number of spontaneous recombinations per second in unit volume, and is expressed by an integral:

$$R_s = \int_0^\infty r_s(\hbar\omega) d(\hbar\omega) = \int_{E_g}^\infty r_s(\hbar\omega) d(\hbar\omega), \quad (1)$$

where  $r_s$  is a spectral function of spontaneous recombination. It can be obtained on the basis of quantum mechanics.

Standard quantum-mechanical calculations (similar to those given in Refs. 24 and 25) lead to the following expression for spectral density of the transition probability with emission of the photon with energy  $E_{\mathbf{k}}$  and arbitrary polarization and arbitrary direction of the wave vector  $\mathbf{k}$ :

$$W_{cv} = \frac{2e^2\sqrt{\epsilon}\hbar\omega}{m_0^2Vc^3\hbar^2} \mathbf{M}^2 [N(E_{\mathbf{k}}) + 1] \delta(E_c - E_v - E_{\mathbf{k}}), \quad (2)$$

where  $\sqrt{\epsilon}$  is the refractive index,  $m_0$  is free electron mass, and

$$\mathbf{M}^2 = \frac{1}{4\pi} \sum_{\lambda=1}^2 \int d\Omega_{\mathbf{k}} |\mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{P}_{cv}|^2,$$

where  $\mathbf{e}_{\mathbf{k}\lambda}$  is the polarization vector of the photon with the momentum  $\mathbf{k}$ . The sum is over two polarizations, the integration is over all photon momentum directions, and  $\mathbf{P}_{cv}$  is the interband transition matrix element at the  $\Gamma$  point of the Brillouin zone.

For spontaneous emission of a photon one has to put the number of photons  $N(E_{\mathbf{k}})$  in the expression (2) equal to zero. Multiplying the probability by the carrier statistical factor  $f(E_c)[1 - f(E_v)]$ ,  $f$  being the Fermi–Dirac function  $f = \{1 + \exp[(E - F)/k_B T]\}^{-1}$  and  $E_c$  and  $E_v$  the electron and hole energies, and integrating over all states of the particles in the conductivity and valence bands, one can obtain an expression for the spectral function  $r_s$ :

$$r_s(\hbar\omega) = \frac{2e^2\sqrt{\epsilon}\hbar\omega}{m_0^2Vc^3\hbar^2} \mathbf{M}^2 \int \frac{2V}{(2\pi)^3} d^3\mathbf{k}_c d^3\mathbf{k}_v f(E_c) \times [1 - f(E_v)] \delta(\mathbf{k}_c - \mathbf{k}_v) \delta(E_c - E_v - \hbar\omega). \quad (3)$$

For nondegenerate semiconductors the Fermi–Dirac functions are reduced to

$$f(E_c) \rightarrow f_c(E_c) = \exp\left(\frac{F - E_c}{k_B T}\right),$$

$$[1 - f(E_v)] \rightarrow f_v(E_v) = \exp\left(\frac{E_v - F}{k_B T}\right).$$

Thus, we obtain

$$r_s(\hbar\omega) = \frac{e^2\sqrt{\epsilon}\hbar\omega}{m_0^2\pi^3c^3\hbar^2} \mathbf{M}^2 \int e^{\frac{E_v - E_c}{k_B T}} \delta(\mathbf{k}_c - \mathbf{k}_v) \times \delta(E_c - E_v - \hbar\omega) d^3\mathbf{k}_c d^3\mathbf{k}_v.$$

First we perform the summation over one of the wave vectors (namely, over  $\mathbf{k}_v$ ) and obtain

$$r_s(\hbar\omega) = \frac{e^2\sqrt{\epsilon}\hbar\omega}{m_0^2\pi^3c^3\hbar^2} \mathbf{M}^2 \int e^{\frac{E_v - E_c}{k_B T}} \delta(E_c - E_v - \hbar\omega) d^3\mathbf{k}. \quad (4)$$

Then, denoting by  $L$  the coefficient in Eq. (4)

$$L = \frac{e^2\sqrt{\epsilon}}{m_0^2\pi^3c^3\hbar^2} \mathbf{M}^2$$

and substituting it in Eq. (1) one can obtain

$$R_s = L \int_{E_g}^\infty \hbar\omega d(\hbar\omega) \int e^{\frac{E_v - E_c}{k_B T}} \delta[E_c(\mathbf{k}) - E_v(\mathbf{k}) - \hbar\omega] d^3\mathbf{k},$$

where the integral is taken over photon energies  $\hbar\omega \geq E_g$ .

The carrier energy dispersion laws in the nitrides are in general nonparabolic. The degree of the nonparabolicity, however, differs in different bands. According to recent spectrum calculations,<sup>18,19</sup> this effect is more prominent for  $\Gamma_7^v(\Gamma_1)$  and  $\Gamma_7^v(\Gamma_6)$  bands, whereas  $\Gamma_7^c$  and  $\Gamma_9^v$  can be considered as parabolic at typical carrier energies of few  $k_B T$ . We will see in the Sec. V that the most important recombination channel corresponds exactly to  $\Gamma_7^c \rightarrow \Gamma_9^v$  transitions due to high density of states in  $\Gamma_9^v$  band. So we can use an anisotropic band energy dispersion law in the parabolic approximation for these bands:

$$E_c = E_c^0 + \frac{\hbar^2}{2} \left( \frac{k_x^2}{m_{ex}} + \frac{k_y^2}{m_{ey}} + \frac{k_z^2}{m_{ez}} \right),$$

$$E_v = E_v^0 - \frac{\hbar^2}{2} \left( \frac{k_x^2}{m_{hx}} + \frac{k_y^2}{m_{hy}} + \frac{k_z^2}{m_{hz}} \right),$$

$$E_c - E_v = E_g + \frac{\hbar^2}{2} \left( \frac{k_x^2}{\mu_x} + \frac{k_y^2}{\mu_y} + \frac{k_z^2}{\mu_z} \right),$$

$$E_g = E_c^0 - E_v^0,$$

where  $\mu_x = (m_{e,x}^{-1} + m_{h,x}^{-1})^{-1}$  is the reduced carrier mass in  $x$  direction, and similarly for  $y$  and  $z$ .

After a scale transformation  $k_a \rightarrow k_a / \sqrt{2\mu_a}$  with the Jacobian  $\sqrt{8\mu_x\mu_y\mu_z}$  and integration in spherical coordinates, we find:

$$R_s = \frac{2\pi}{\hbar^3} L \sqrt{8\mu_x\mu_y\mu_z} \int_{E_g}^\infty e^{-\frac{\hbar\omega}{k_B T}} \hbar\omega (\hbar\omega - E_g)^{1/2} d(\hbar\omega).$$

This integral is reduced by elementary transformations to two  $\Gamma$  functions:  $\Gamma(3/2) = \sqrt{\pi}/2$ ;  $\Gamma(5/2) = 3\sqrt{\pi}/4$ , and thus we obtain, finally, the following expression for the spontaneous radiative recombination rate:

$$R = \frac{\sqrt{\epsilon}e^2}{m_0^2c^3\hbar^2} \mathbf{M}^2 \left[ \frac{2k_B T}{\pi\hbar^2} \right]^{3/2} \sqrt{\mu_x\mu_y\mu_z} E_g \left[ 1 + \frac{3k_B T}{2E_g} \right] e^{-E_g/k_B T}. \quad (5)$$

Defining the radiative recombination coefficient  $B$  according to the equality

$$R = Bnp,$$

where  $n$  and  $p$  are carrier concentrations and using the well-known expression for concentration of electrons and holes in a nondegenerate semiconductor:

$$np = 4(m_e m_h)^{3/2} \left( \frac{k_B T}{2\pi\hbar^2} \right)^3 \exp\left(-\frac{E_g}{k_B T}\right),$$

one comes to the following expression:

$$B = \frac{\sqrt{\epsilon} e^2}{m_0^2 c^3 \hbar^2} \mathbf{M}^2 \left[ \frac{2\pi\hbar^2}{k_B T} \right]^{3/2} \frac{1}{(\bar{m}_x \bar{m}_y \bar{m}_z)^{1/2}} E_g(T) \times \left[ 1 + \frac{3k_B T}{2E_g(T)} \right], \quad (6)$$

where  $\bar{m}_x = m_{e,x} + m_{h,x}$  and so on. One can see from this equation that for the calculation of the radiative recombination coefficient  $B$  it is necessary to know the matrix element  $\mathbf{M}^2$  and the temperature dependence of  $E_g$ .

#### IV. TEMPERATURE AND ALLOY COMPOSITION DEPENDENCE OF THE ENERGY GAP

In many semiconductors, including the nitrides, the empirical Varshni formula<sup>26</sup> approximates well the observed temperature dependence of the gap:

$$E_g(T) = E_g(0) - \frac{\gamma T^2}{T + \beta},$$

where  $\gamma$  and  $\beta$  are parameters. Their values for nitride thin films were found in Refs. 15–17:

AlN:  $E_g(300 \text{ K}) = 6.026 \text{ eV}$ ,  
 $\gamma = 1.799 \times 10^{-3} \text{ eV/K}$ ,  $\beta = 1462 \text{ K}$ ,  
 GaN:  $E_g(0) = 3.427 \text{ eV}$ ,  
 $\gamma = 0.939 \times 10^{-3} \text{ eV/K}$ ,  $\beta = 772 \text{ K}$ ,  
 InN:  $E_g(300 \text{ K}) = 1.970 \text{ eV}$ ,  
 $\gamma = 0.245 \times 10^{-3} \text{ eV/K}$ ,  $\beta = 624 \text{ K}$ .

There are indications (see, for example, Ref. 17 for GaN) showing that the Varshni formula parameters depend on the fabrication method of the nitride films of the same wurtzite structure. In our calculation we have taken the values of these parameters found in single-crystal films produced by MBE. For the gap width in the binary alloys of the components A and B, we used the expression<sup>27</sup>

$$E_g^{(AB)}(x, T) = [xE_g^{(A)}(T) + (1-x)E_g^{(B)}(T) - dx(1-x)]$$

with  $d = 1.3$  for  $\text{Ga}_x\text{Al}_{1-x}\text{N}$ ,  $d = 2.6$  for  $\text{In}_x\text{Al}_{1-x}\text{N}$  and  $d = 0.6$  for  $\text{Ga}_x\text{In}_{1-x}\text{N}$  (the data from works<sup>3,10,28,29</sup>).

#### V. CALCULATION OF THE INTERBAND MATRIX ELEMENT

To find the interband transition matrix elements, we made use of the formula for the absorption coefficient that also contains  $\mathbf{P}_{cv}$  (see, for example, Ref. 30):

$$\alpha(\hbar\omega) = b \sqrt{\hbar\omega - E_g},$$

$$b = \frac{2e^2 \sqrt{8\mu_x \mu_y \mu_z}}{\hbar^2 m_0^2 \sqrt{\epsilon} c E_g} \frac{1}{2\pi} \sum_{\lambda=1}^2 \int d\phi |\mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{P}_{cv}|^2, \quad (7)$$

where the integration is over all polarization directions in the plane perpendicular to the incident photon beam, as distinct from the integration over all photon wave vector directions in the calculations of the spontaneous radiative recombination rate  $R$ . Extracting the coefficient  $b$  from the measured frequency dependence of  $\alpha$ , one can find  $\mathbf{P}_{cv}$  components in the  $xy$  plane assuming that the light beam was perpendicular to the film surface. However, this method does not allow us to find  $P_{cv,z}$ . This is not important for the transitions from  $\Gamma_7^c$  to  $\Gamma_9^v$  as in InN and GaN (see selection rules above), but in AlN where crystal split-off  $\Gamma_7^v$  band lies higher than  $\Gamma_9^v$ , this may cause problems. We still calculated  $B$  for AlN, assuming that  $\Gamma_7^v - \Gamma_9^v$  splitting in AlN is rather small [ $\Delta E_{\text{split}} \approx 20 \text{ meV}$  (Ref. 31)], and the hole population of  $\Gamma_9^v$ , and hence the transition probability to this band, may be higher than that in  $\Gamma_7^v$  due to much higher density of states in  $\Gamma_9^v$ . The values of  $P_{cv,x} = P_{cv,y}$  found this way together with other material parameters used in our calculations are listed below:

AlN:  $P_{cv,x} = 9.5 \times 10^{-20} \text{ g cm/s}$ ,  $\sqrt{\epsilon} = 2.15$ ,  
 $m_e = 0.27$ ,  $m_{h,z} = 3.68$ ,  $m_{h,xy} = 6.33$  (Ref. 18);  
 GaN:  $P_{cv,x} = 9.2 \times 10^{-20} \text{ g cm/s}$ ,  $\sqrt{\epsilon} = 2.67$ ,  
 $m_e = 0.18$ ,  $m_{h,z} = 1.76$ ,  $m_{h,xy} = 1.69$  (Ref. 19);  
 InN:  $P_{cv,x} = 11.9 \times 10^{-20} \text{ g cm/s}$ ,  $\sqrt{\epsilon} = 2.1$ ,  
 $m_e = 0.11$ ,  $m_{h,z} = 1.56$ ,  $m_{h,xy} = 1.68$  (Ref. 19).

The photon energy dependence of the absorption coefficient was taken from Ref. 10 for AlN, Ref. 6 for GaN, and Ref. 9 for InN. The method we used to obtain the matrix element of the radiative transition includes the assumption that all contributions of defects in the nitride thin films (such as nitrogen vacancies, oxygen impurities, etc.) to the absorption spectrum are noticeable only below and around the fundamental absorption edge. Deeper in the region of the band–band transitions where the fundamental absorption greatly increases, these contributions can be neglected. This proposition is consistent with results of experimental works (for example, see Refs. 5 and 10). So we extracted the matrix element values from the experimental data using just this area of maximum absorption.

The values of the matrix element found this way are in a good agreement with those obtained in other works.<sup>32,33</sup> In the next section we will show that the calculated exciton lifetimes based on these values are also in agreement with available experimental data.

Now let us turn to the calculations of the interband radiative recombination rate of free carriers in the nitride semiconductor alloys. The values of the matrix elements and effective masses in the alloys were found using a linear interpolation between the values in the alloy components. The calculated temperature dependence of the radiative recombination coefficient  $B$  is shown in Figs. 1–3. One can see

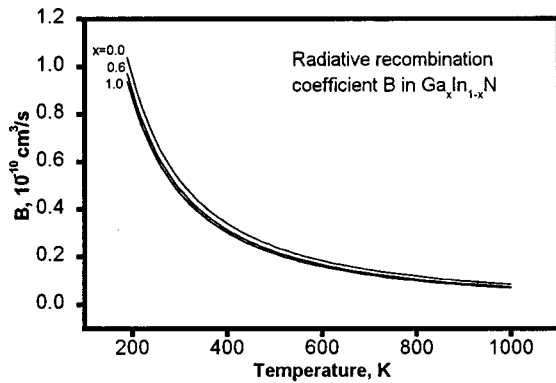


FIG. 1. The radiative recombination coefficient  $B$  vs temperature  $T$  and alloy composition  $x$  for  $\text{Ga}_x\text{In}_{1-x}\text{N}$ .

that  $B$  is the highest in InN and the compositions close to it, and the lowest in AlN and the alloys close to this material. Figure 1 depicts that the radiative recombination rate of  $\text{Ga}_x\text{In}_{1-x}\text{N}$  is almost the alloy composition independent. The effective masses and refractive indices have close values in these materials, and the difference in the band gap values is just compensated by the change in the matrix element [see Eq. (6)].

The radiative recombination coefficient and the radiative free carrier lifetime defined as  $\tau_r^{-1} = Bn$  are presented in Table I for different materials and alloys. One can see that in similar conditions the carrier lifetime due to free carrier radiative recombination even in AlN is higher than in GaAs. Please note that our  $B$  coefficient of GaN is shown to be nearly twice as large as one from Ref. 34. This fact can be explained by using the recent effective masses values we took from new energy spectrum calculations.<sup>18,19</sup>

## VI. LIFETIMES OF EXCITONS IN NITRIDES

With the above matrix elements we can also calculate the radiative lifetimes of free excitons in the nitride semiconductors and compare them with the data of experimental works.<sup>22,35</sup>

It is known that exciton annihilation dominate the carrier radiative recombination in the nitrides. This fact was confirmed by observations of photoluminescence in this material.<sup>22</sup> The predominance of exciton radiative recombina-

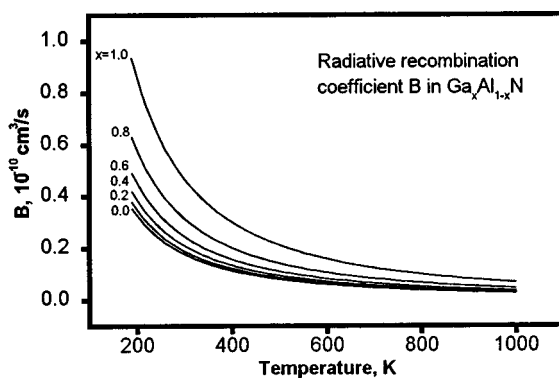


FIG. 2. The radiative recombination coefficient  $B$  vs temperature  $T$  and alloy composition  $x$  for  $\text{Ga}_x\text{Al}_{1-x}\text{N}$ .

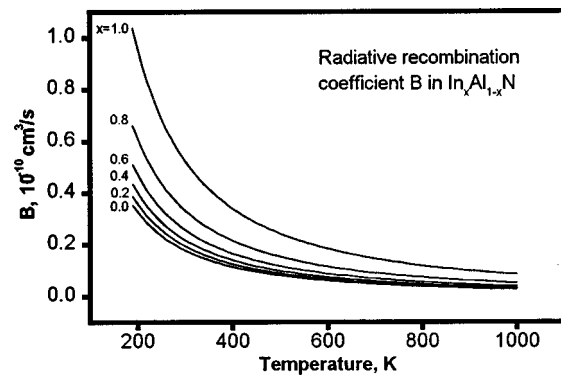


FIG. 3. The radiative recombination coefficient  $B$  vs temperature  $T$  and alloy composition  $x$  for  $\text{In}_x\text{Al}_{1-x}\text{N}$ .

tion is the characteristic property of wide-gap direct band semiconductors (such as, for example, CdS and GaP<sup>36</sup>). The creation time of free exciton in them is an order of magnitude less than the radiative lifetime of free carriers.<sup>37</sup>

There are three valence band subbands (see Sec. II) in the electron spectra of the nitrides AlN, GaN, InN, which give rise to three different free excitons. They are clearly observable in GaN (see Refs. 4, 22, and 38). We calculated the temperature dependence of the free exciton lifetime for the brightest line in photoluminescence spectrum of the nitride semiconductors. This line corresponds to the free exciton with a heavy hole from  $\Gamma'_9$  subband.

In direct semiconductors the free exciton radiative recombination rate can be derived from the probability of the photon emission under exciton recombination<sup>24</sup>

$$W_{ph} = \frac{2}{3} \frac{e^2 \sqrt{\epsilon(E_g - E_x)}}{m_0^2 c^3 \hbar^2} |\mathbf{P}_{cv}|^2 \left[ \frac{2\pi \hbar^2}{k_B T} \right]^{3/2} \frac{1}{(\bar{m}_x \bar{m}_y \bar{m}_z)^{1/2}} \times |F(0)|^2 \delta_{\mathbf{k}}, \quad (8)$$

where  $E_x$  is the exciton binding energy:

$$E_x = \frac{m_0 e^4}{2 \hbar^2} \frac{\mu}{\epsilon^2},$$

$F(r)$ , an  $s$ -like envelope function, is a solution of the effective mass equation for the free exciton ground state

TABLE I. The radiative recombination coefficient  $B$  and an average lifetime  $\tau_r$  of carriers (at  $T = 300$  K and  $n = 10^{18}$   $\text{cm}^{-3}$ ).

	$B$ ( $10^{-10}$ $\text{cm}^3/\text{s}$ )	$\tau_r$ (ns)
AlN	0.18	55
InN	0.52	19
$\text{Ga}_{0.6}\text{In}_{0.4}\text{N}$	0.49	20
GaN	0.47	21
$\text{Ga}_{0.8}\text{Al}_{0.2}\text{N}$	0.31	32
$\text{In}_{0.6}\text{Al}_{0.4}\text{N}$	0.26	38
GaN <sup>a</sup>	0.24	
GaAs <sup>b</sup>	7.2	1.3

<sup>a</sup>The data were taken from Ref. 34.

<sup>b</sup>The data were taken from Ref. 30.

TABLE II. Parameters of free excitons in the nitrides.

	$\epsilon$	$E_x$ (meV)	$a_x$ (Å)
InN	9.3	16	49.2
GaN	9.5	20	38.7
AlN	8.5	49	17.3

$$F(r) = \sqrt{\frac{V}{\pi a_x^3}} \exp\left(-\frac{r}{a_x}\right),$$

$a_x = a_B \epsilon / \mu$  being the exciton Bohr radius, and  $V$  is the system volume. To take the anisotropy of the electron and hole spectra into account, we take an averaged value for the exciton reduced mass:<sup>22</sup>  $\mu = [(2/3)\mu_{xy}^{-1} + (1/3)\mu_z^{-1}]^{-1}$ , where the perpendicular components are  $\mu_{xy} = (m_{e,xy}^{-1} + m_{h,xy}^{-1})^{-1}$ , and the parallel one is  $\mu_z = (m_{e,z}^{-1} + m_{h,z}^{-1})^{-1}$ . We used the electron mass values presented above in Sec. V.

Equation (8), multiplied by the Boltzmann distribution of excitons (valid in our case) and then integrated over all exciton wave vectors  $\mathbf{k}$ , gives an expression for the radiative recombination probability of one free exciton per unit time:

$$B_x = \frac{2}{3} \frac{\sqrt{\epsilon} e^2}{m_0^2 c^3 \hbar^2} |\mathbf{P}_{cv}|^2 \left[ \frac{2\pi\hbar^2}{k_B T} \right]^{3/2} \frac{1}{(\bar{m}_x \bar{m}_y \bar{m}_z)^{1/2}} (E_g - E_x) \frac{1}{\pi a_x^3}. \quad (9)$$

Finally,  $\tau_x = 1/B_x$  gives the lifetime of excitons

$$\tau_x = K T^{3/2},$$

where a coefficient  $K$  derived from Eq. (9) is

$$K [s/K^{3/2}] = 0.78 \times 10^{-16} \frac{(\bar{m}_x \bar{m}_y \bar{m}_z)^{1/2} a_x^3}{\sqrt{\epsilon} |\mathbf{P}_{cv}|^2 (E_g - E_x)}. \quad (10)$$

Here all fundamental constants were substituted, and  $a_x$  is measured in Å, energies are in eV, and the matrix element  $|\mathbf{P}_{cv}|^2$  is in  $[10^{-38} \text{ g erg}]$ . The exciton parameters can be calculated now on the basis of the data presented above and in Table II.

Our results for the coefficient  $K$  and exciton lifetimes at  $T = 60$  and  $300$  K are tabulated in Table III. When this lifetimes are compared with those of free carriers (see Table I), it is apparent that the lifetimes of free excitons are nearly one order of magnitude less than the radiative lifetimes of free carriers in the nitrides at the same temperature. We point out a good agreement of our values for the exciton lifetimes in GaN with experimental data in Refs. 22, 33–35. We could not find experimental data on the exciton lifetimes in other nitrides.

## VII. CONCLUSION

We calculated radiative recombination coefficients for three nitride semiconductors and their binary alloys. To do it, we extracted values of the interband matrix elements from the absorption data. The matrix elements do not differ considerably in these semiconductors, and the prominent differ-

TABLE III. The lifetimes of free excitons in the nitrides and  $K$  factor for the temperature dependence of radiative lifetime of free excitons.

	$K$ (ps/K <sup>3/2</sup> )	$\tau_x$ (ns)	$\tau_x$ (ns)
		$T = 60$ K	$T = 300$ K
InN	1.4	0.65	7.3
GaN	0.73	0.34	3.8
AlN	0.46	0.21	2.4
GaN	0.71 <sup>a</sup>	0.35 <sup>b</sup>	3.5 <sup>c</sup>

<sup>a</sup>The data were taken from Ref. 34.

<sup>b</sup>The data were taken from Refs. 22 and 35.

<sup>c</sup>The data were taken from Ref. 33.

ence between the recombination coefficients is connected also with the difference in the band gap values and carrier masses.

We estimated radiative lifetimes of excitons in the nitride semiconductors. Our results are in a good agreement with the data presented in other works.<sup>22,33–35</sup>

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